PROBLEM SOLVING

BACK

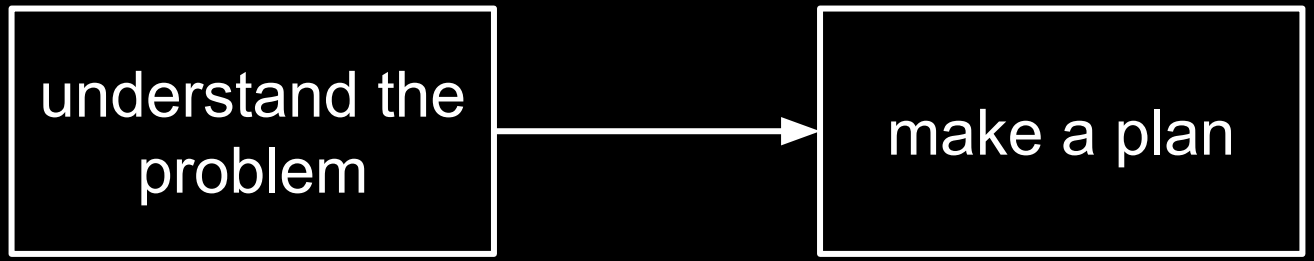
Pólya's approach to problem-solving

image source: http://doi.org/10.3932/ethz-a-000099441

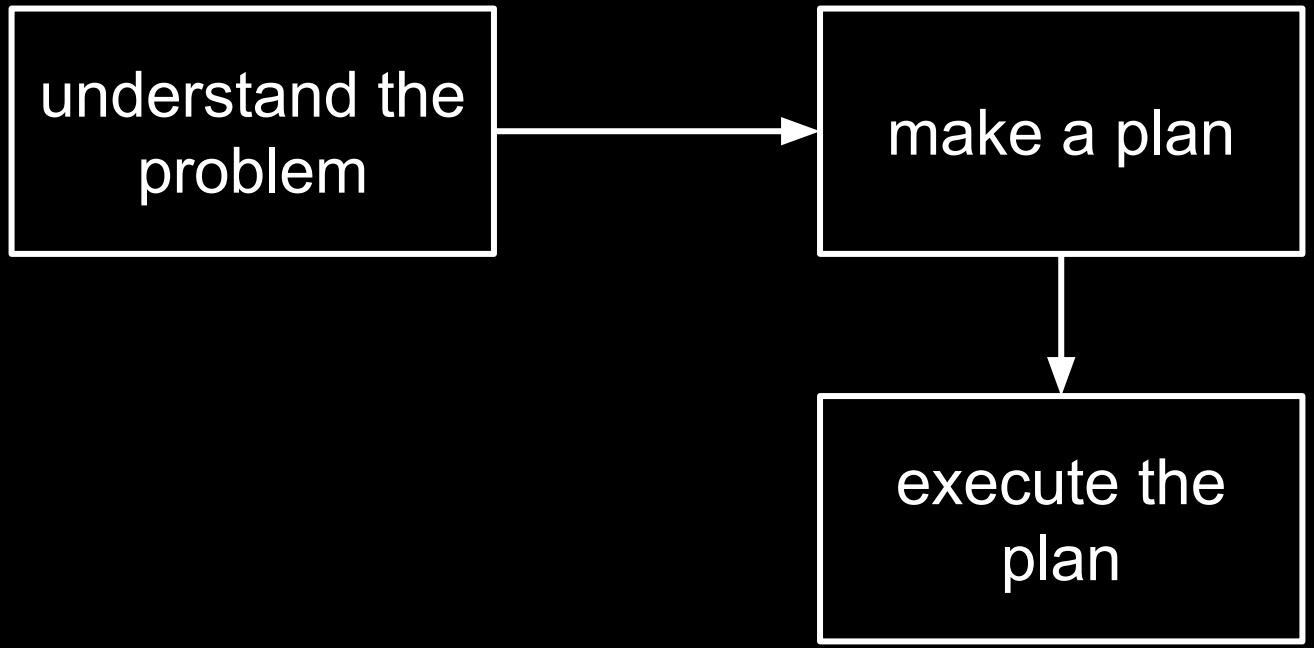
Pólya's approach to problem-solving

understand the problem

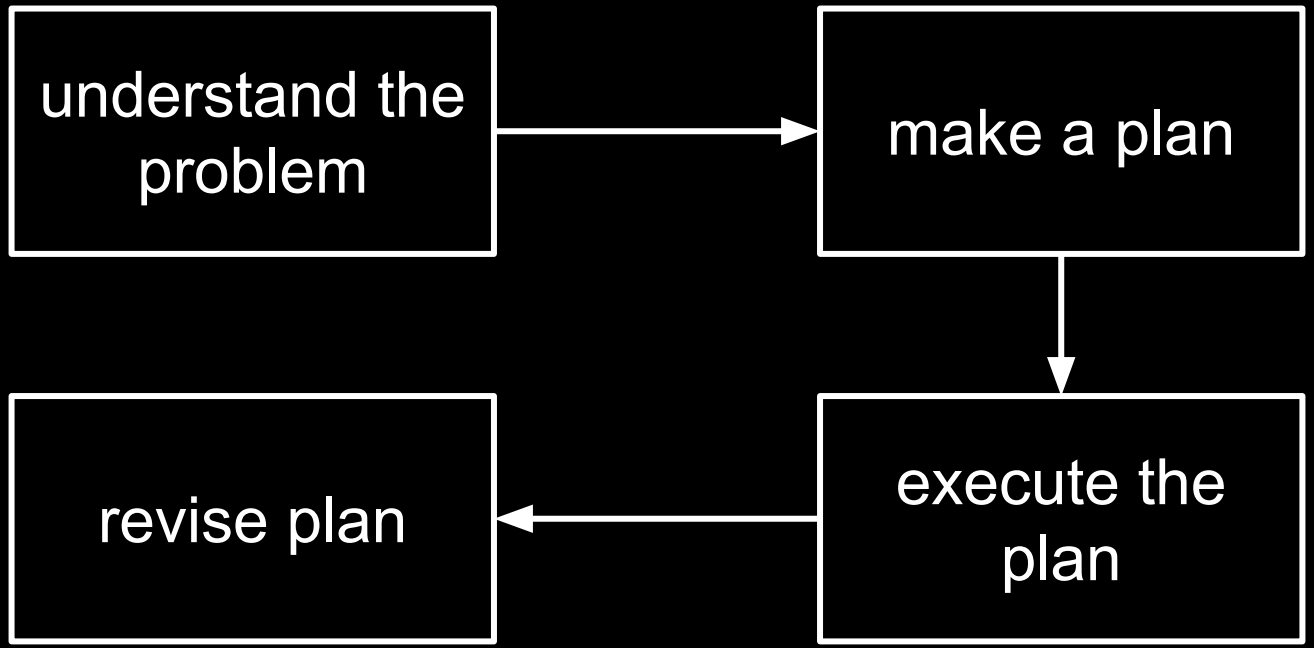
Pólya's approach to problem-solving



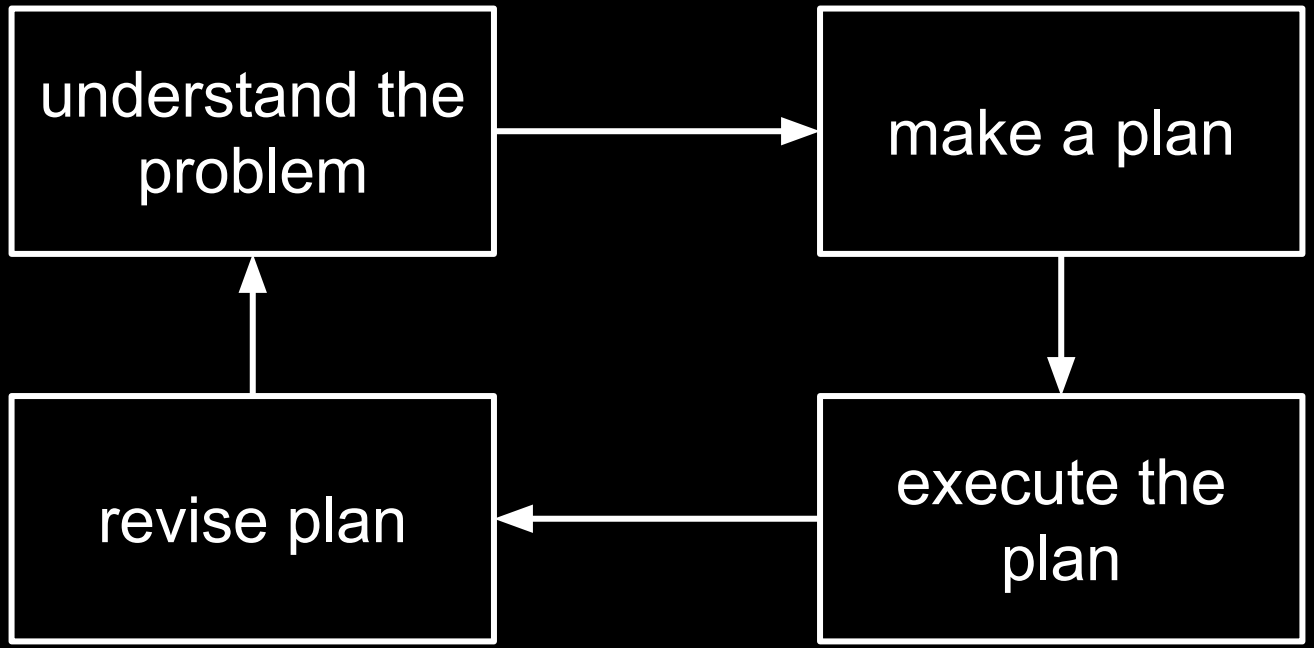
Pólya's approach to problem-solving



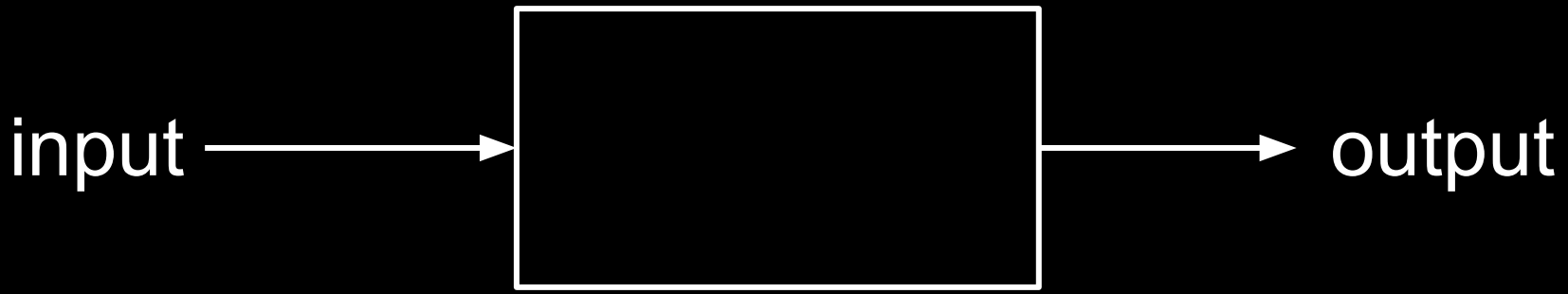
Pólya's approach to problem-solving



Pólya's approach to problem-solving

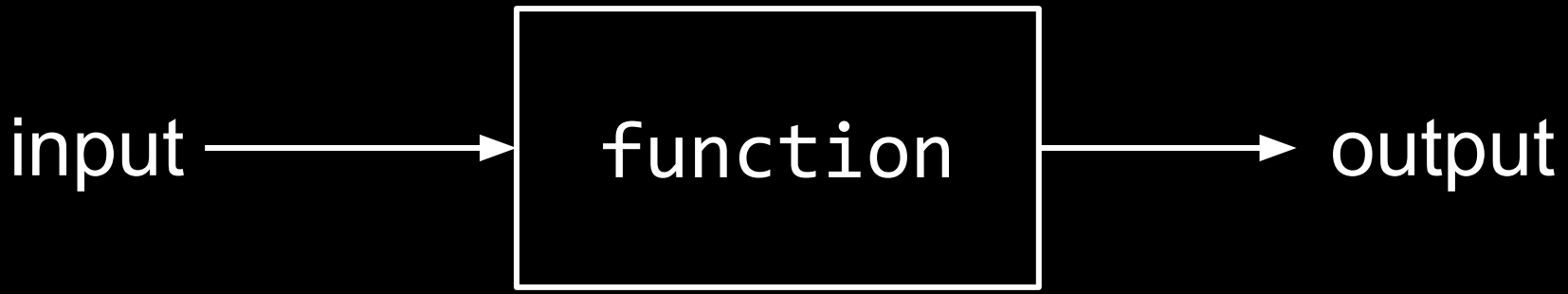


a model to represent problems

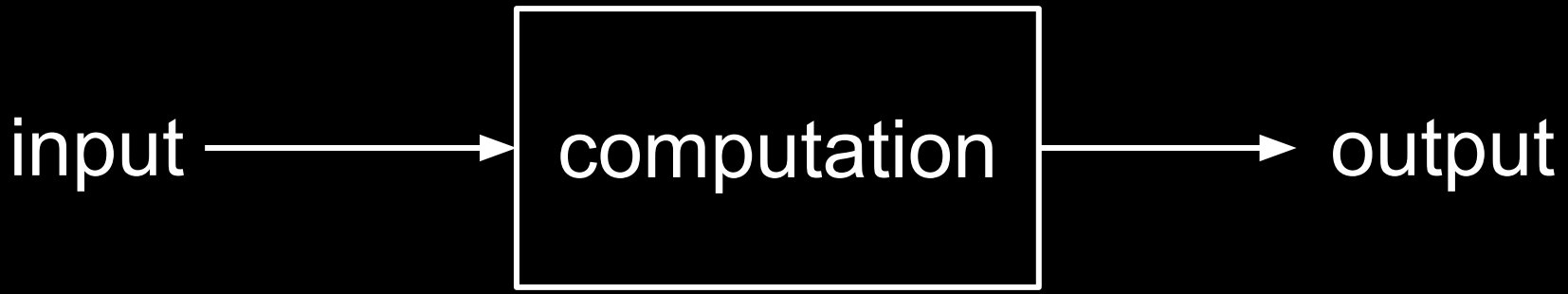


a model to represent problems

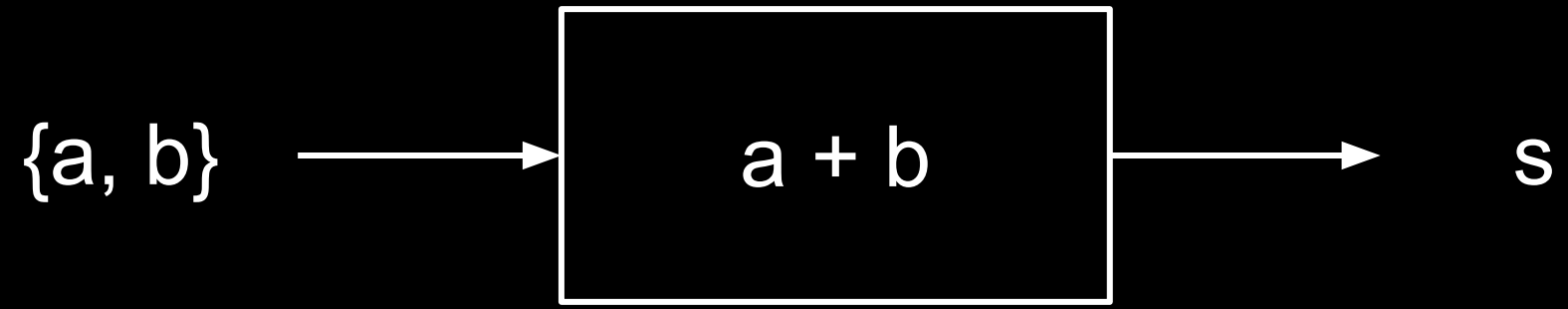
a model to represent problems

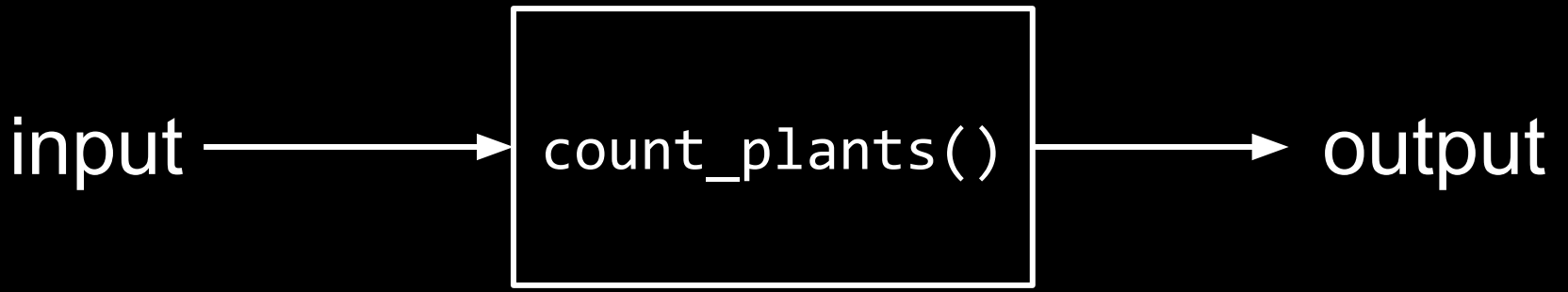


a model to represent problems



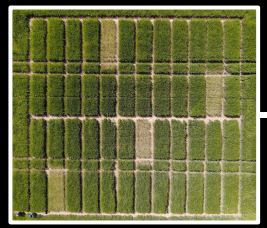
the rainbow experiment as an input - processing - output - problem


{a, b}
a + b
s


input
count_plants()
output

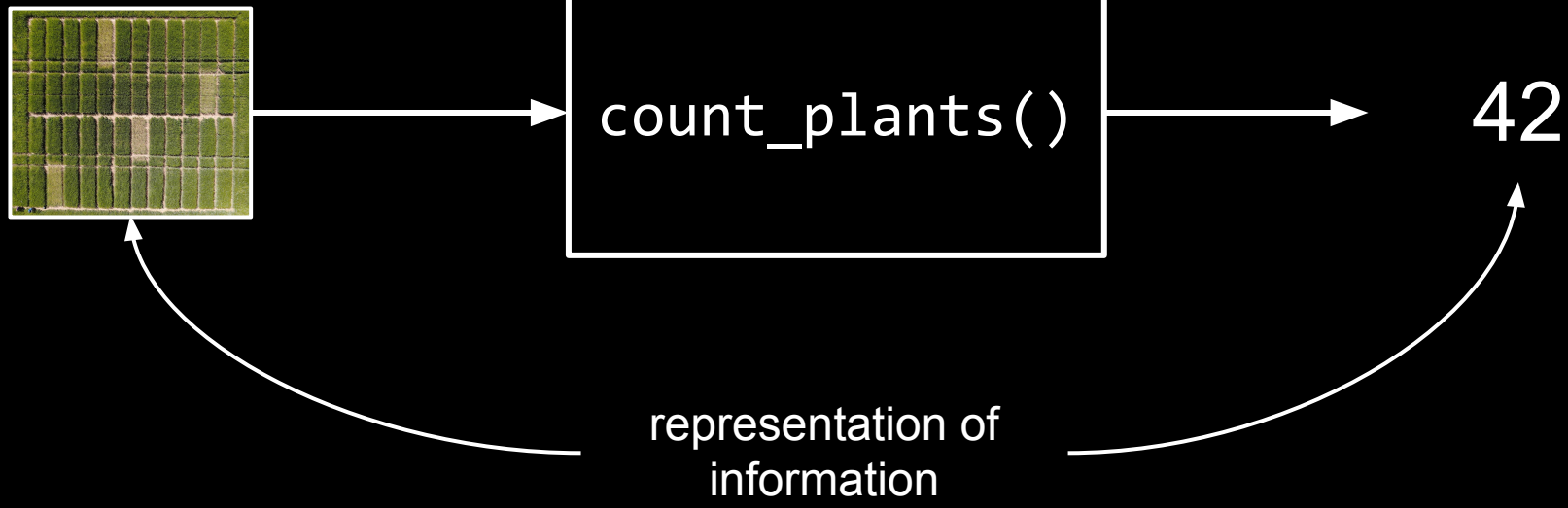
`count_plants()`

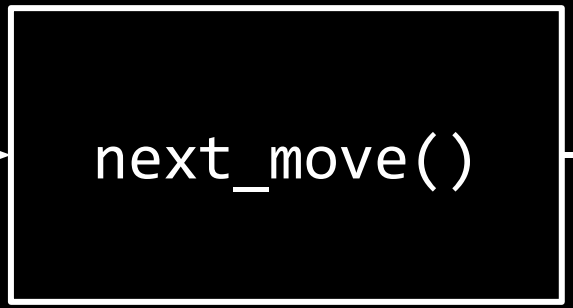
output



```
count_plants()
```

42


processing of
information
count_plants()
42
representation of
information



E2 → E4

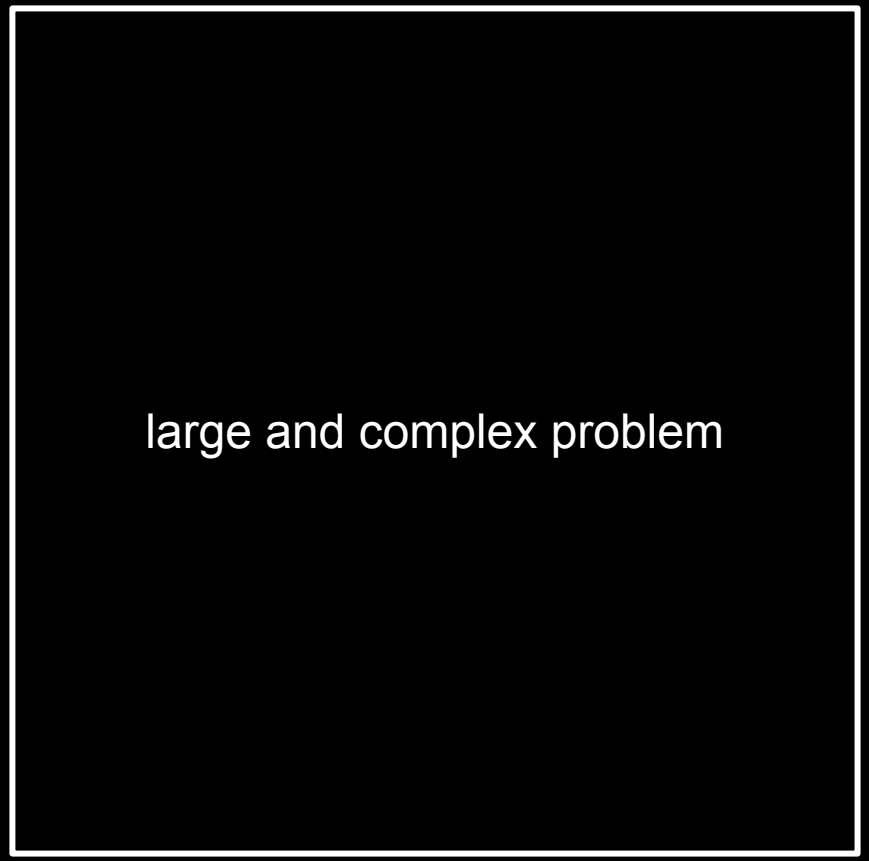
1: 0R	9: 0P		57: 1R
2: 0N	10: 0P		58: 1N
3: 0B	11: 0P		59: 1B
4: 0K	12: 0P		60: 1K
5: 0Q	13: 0P	…	61: 1Q
6: 0B	14: 0P		62: 1B
7: 0N	15: 0P		63: 1N
8: 0R	16: 0P		64: 1R

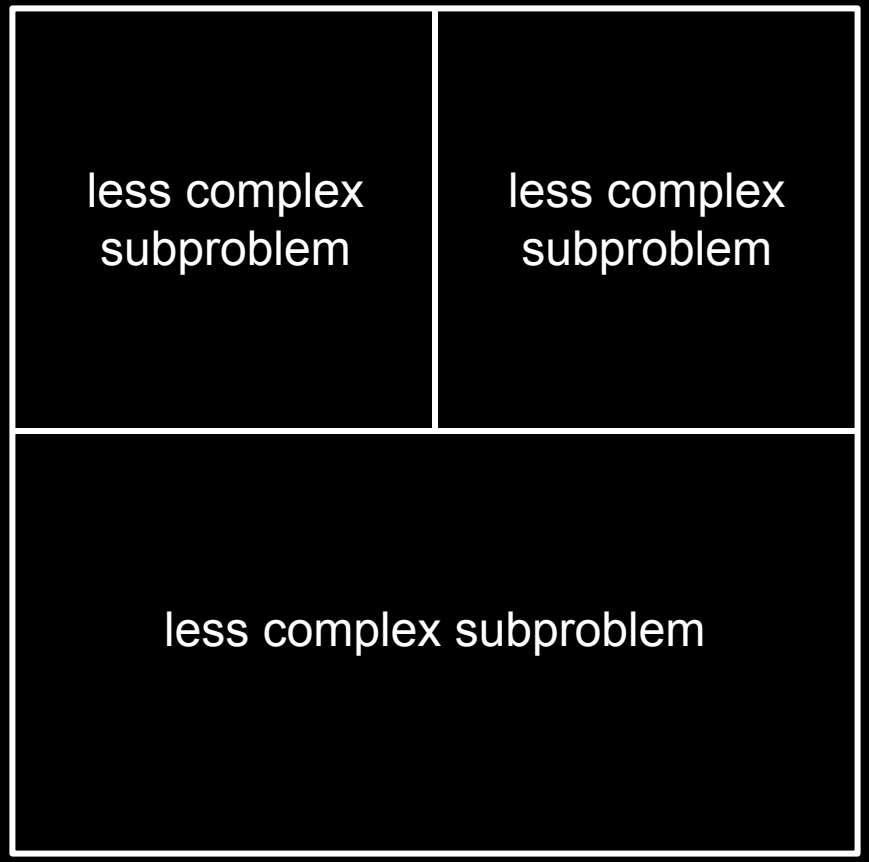
representation of information

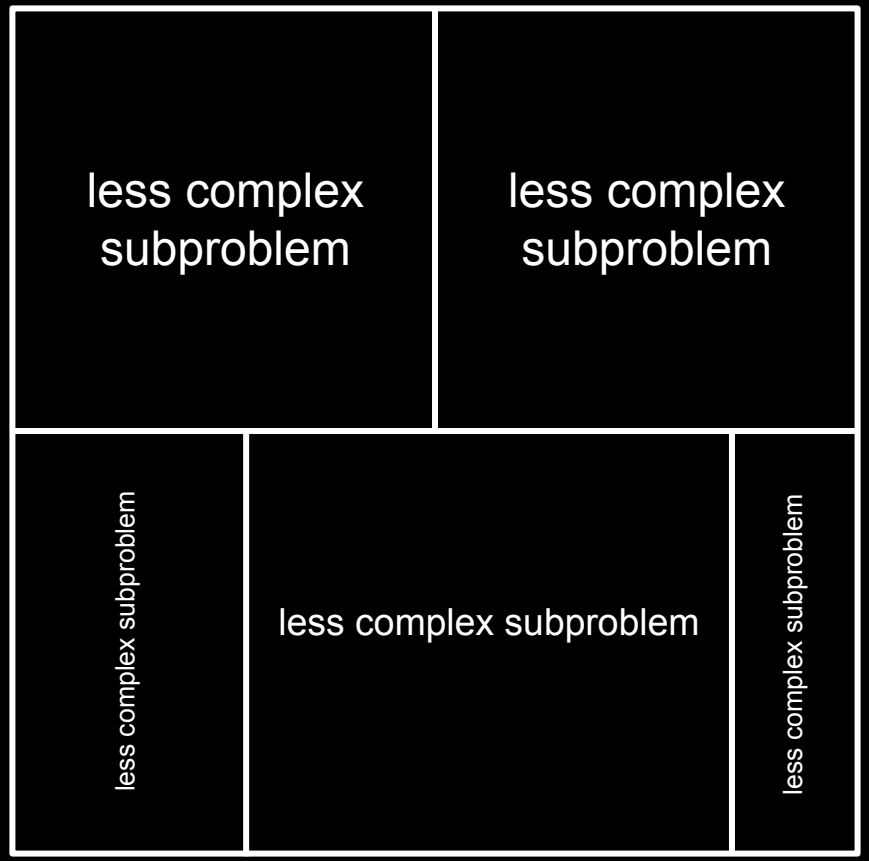

input
ChatGPT
output

problem solving strategies

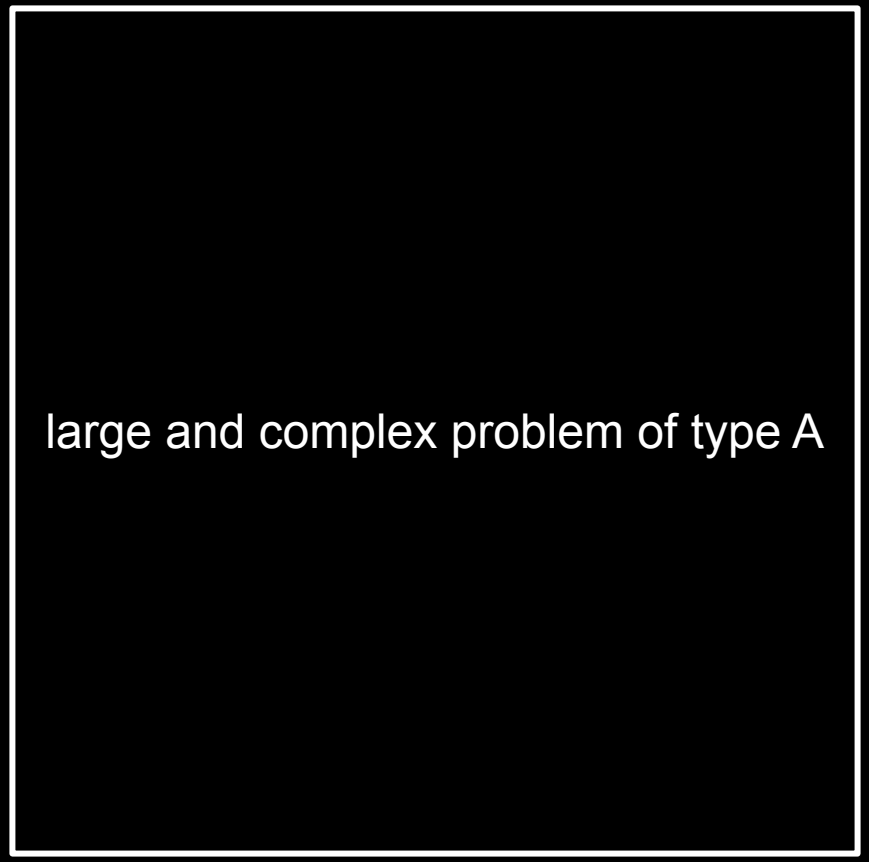
problem decomposition

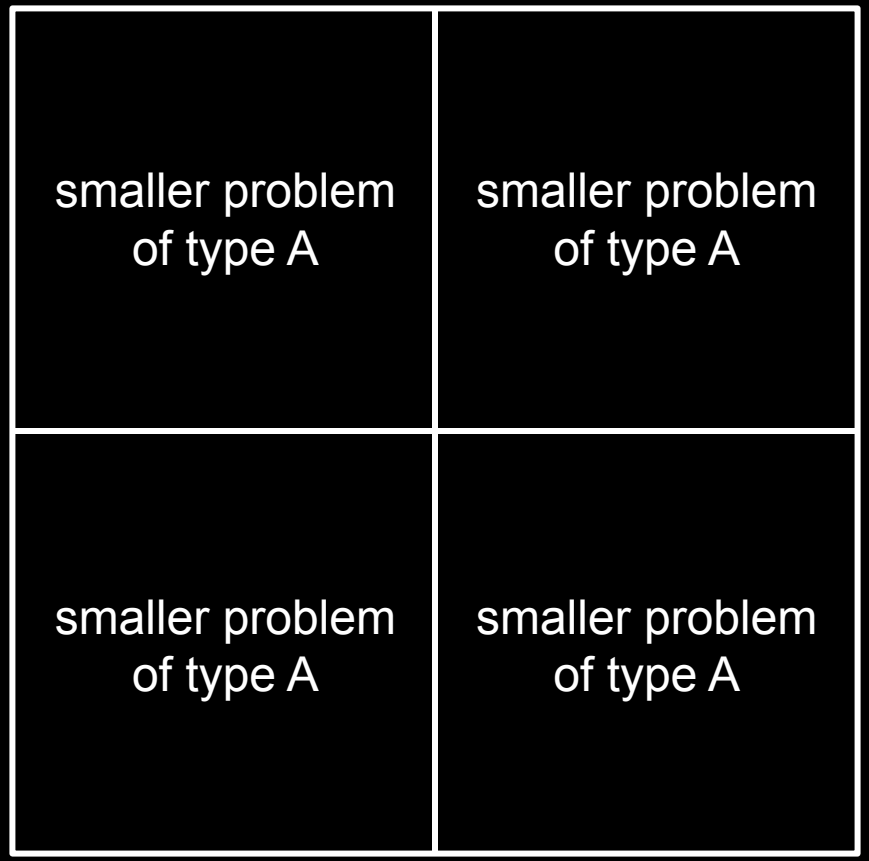

large and complex problem

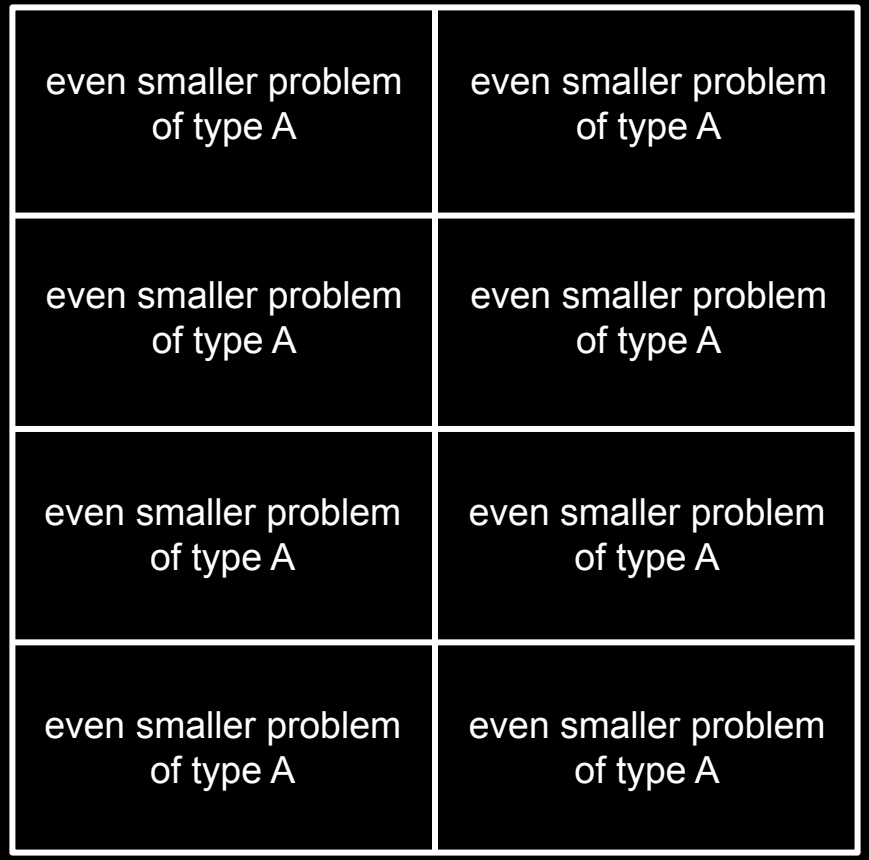

less complex subproblem
less complex subproblem
less complex subproblem


less complex subproblem
less complex subproblem
less complex subproblem
less complex subproblem
less complex subproblem

divide and conquer


large and complex problem of type A


smaller problem of type A
smaller problem of type A
smaller problem of type A
smaller problem of type A


even smaller problem of type A
even smaller problem of type A
even smaller problem of type A
even smaller problem of type A
even smaller problem of type A
even smaller problem of type A
even smaller problem of type A
even smaller problem of type A


sorted list +
element
search()
yes / no

is 67 a prime number?

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

linear search

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

~~2,~~ 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

~~2~~, ~~3~~, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

~~2,~~ ~~3,~~ ~~5,~~ 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

linear search

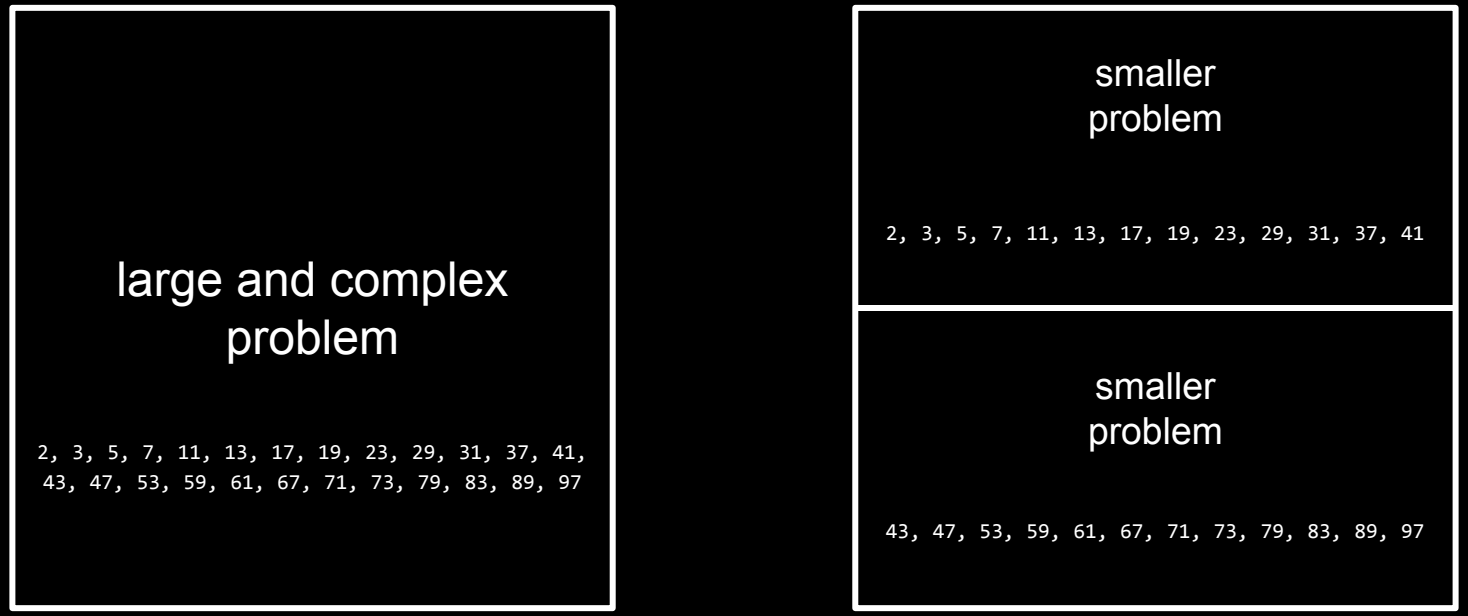
~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

19 steps… can’t we do better?

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, 71, 73, 79, 83, 89, 97

large and complex problem

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97


large and complex problem
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
smaller problem
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41
smaller problem
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

```
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

binary search

67 != 41

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

67 > 41

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

67 != 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, ~~71~~, 73, 79, 83, 89, 97

67 != 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, 59, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 < 71

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
43, 47, 53, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 != 59

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 > 59

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, 61, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 = 67

binary search

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

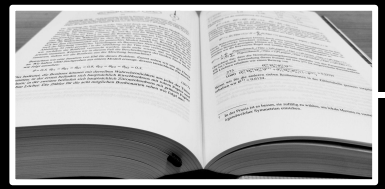
67 = 67

3 splits → much better

~~2~~, ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, ~~19~~, ~~23~~, ~~29~~, ~~31~~, ~~37~~, ~~41~~,
~~43~~, ~~47~~, ~~53~~, ~~59~~, ~~61~~, 67, ~~71~~, ~~73~~, ~~79~~, ~~83~~, ~~89~~, ~~97~~

67 = 67

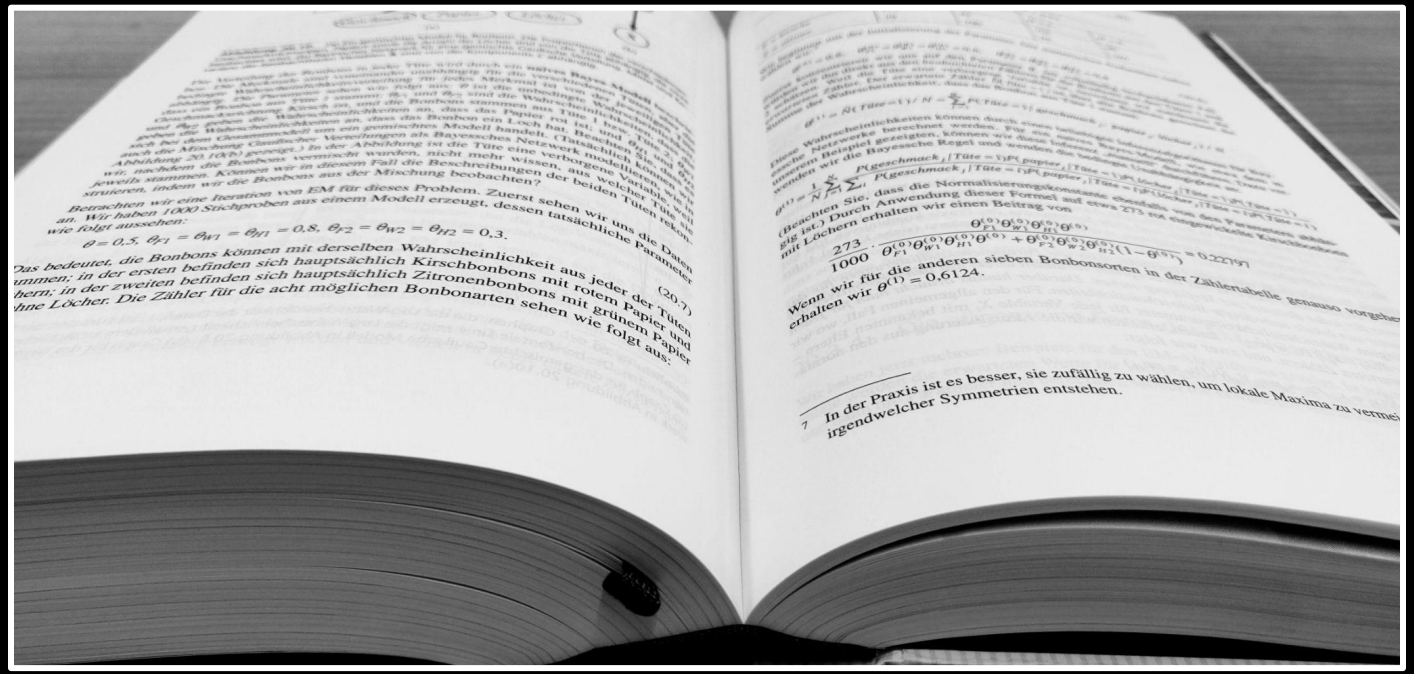
how efficient are linear and binary search in general?



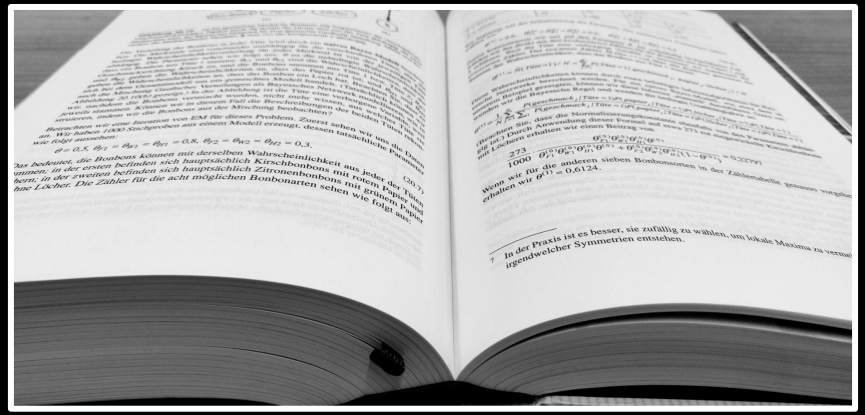
`count_words()`

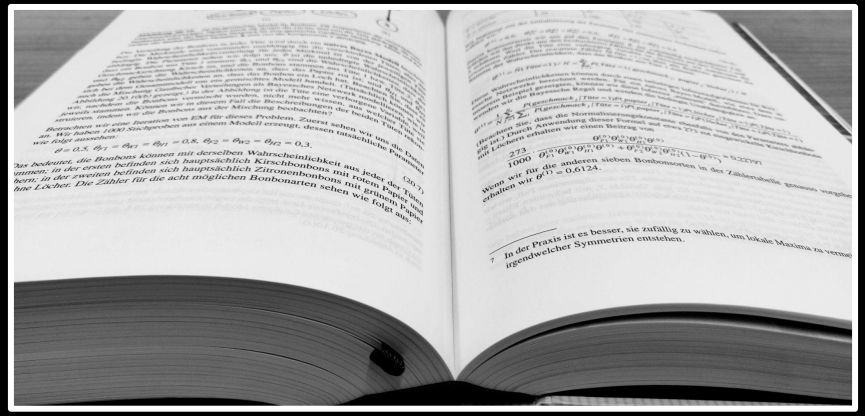
word count

how many words are in the book?

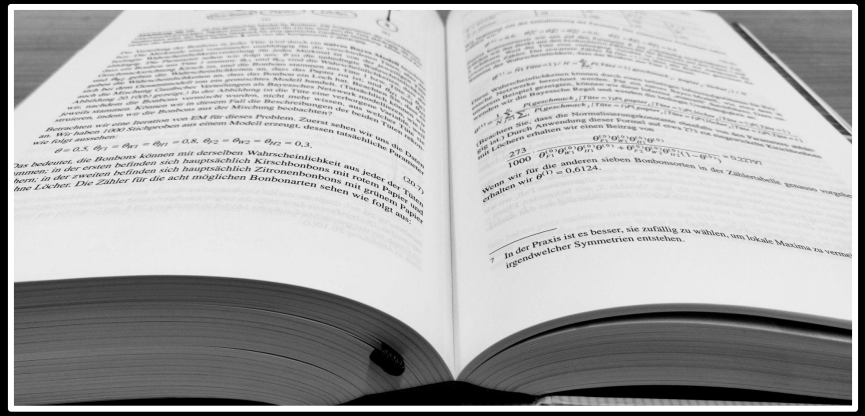


strategies, anyone?



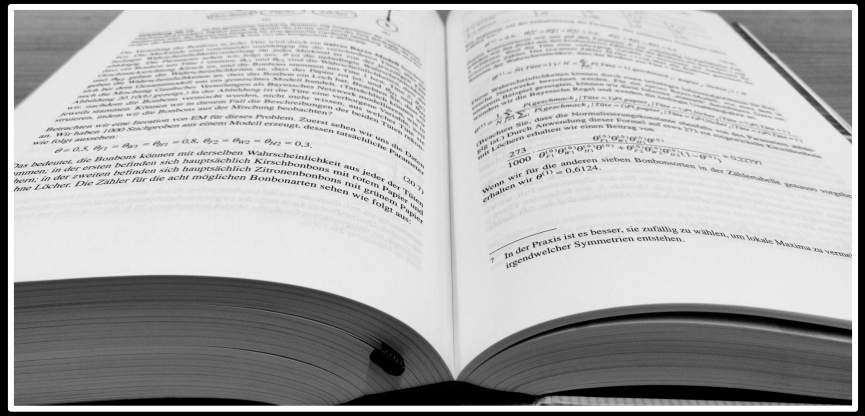


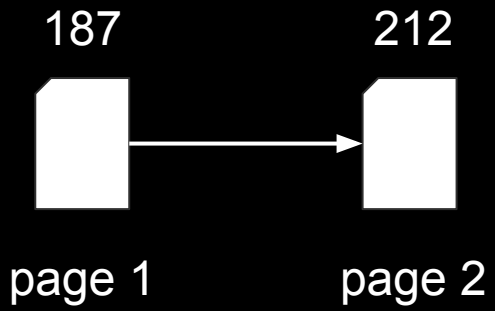
page 1



187

page 1

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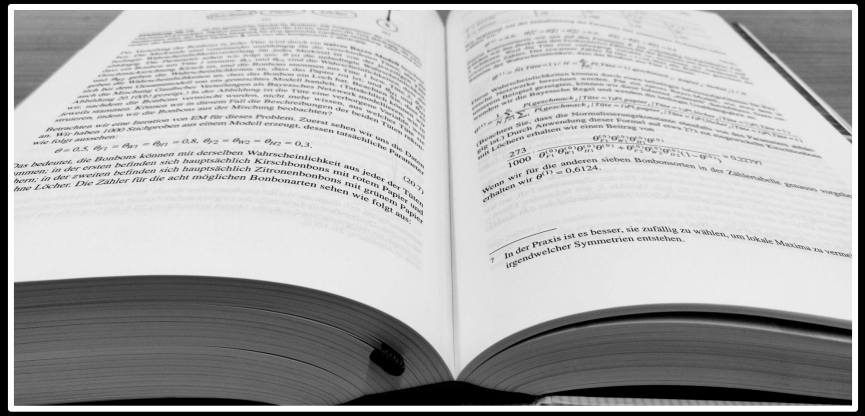
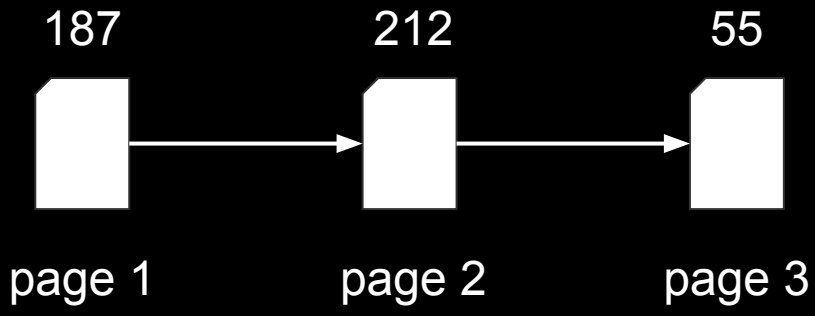
212
page 1
page 2

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page 1
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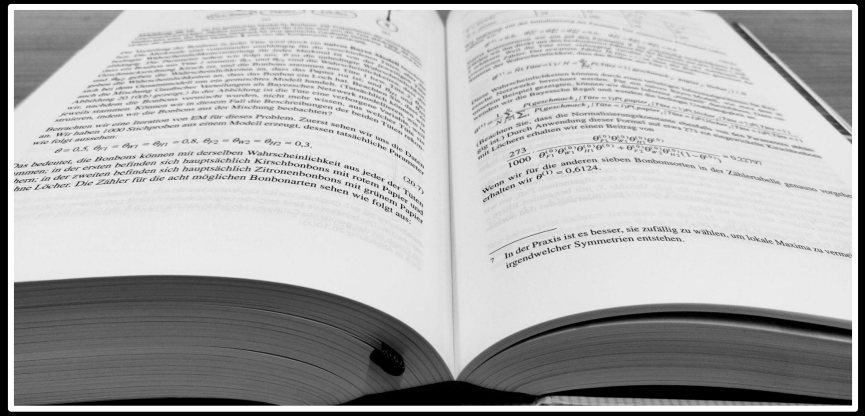
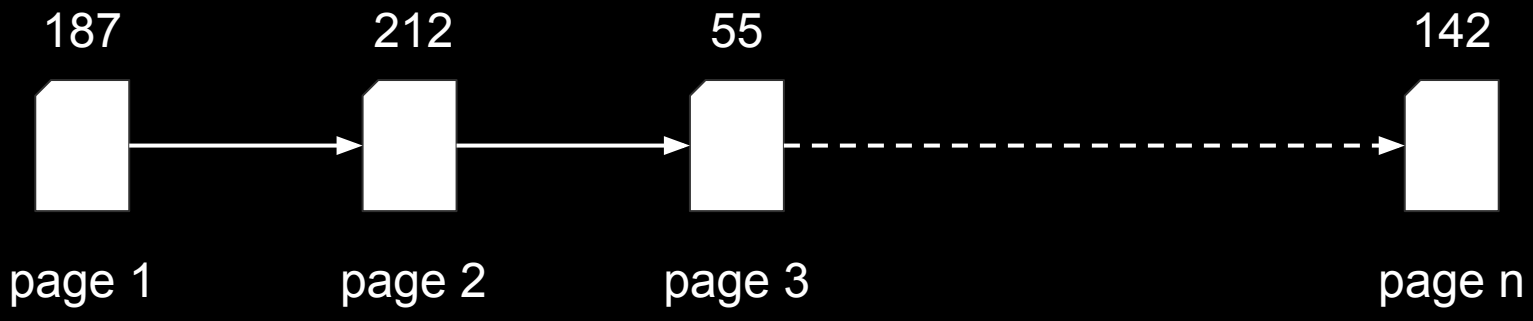
page 3




187
212
55
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page 1
page 2
page 3
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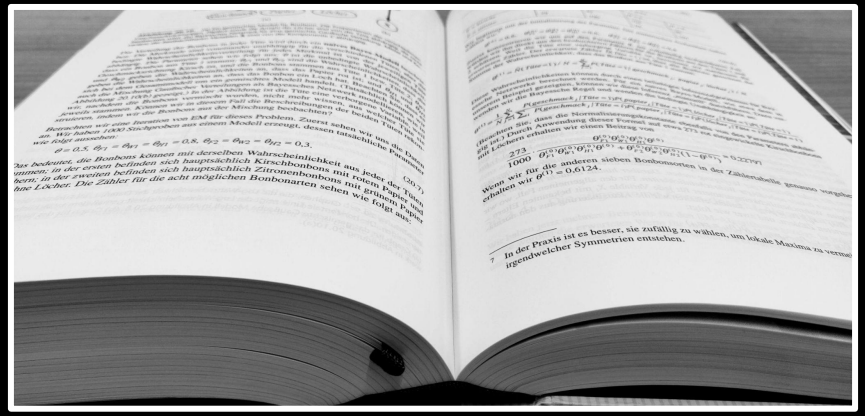


n = 1327 pages

Ø 2:23 minutes per page

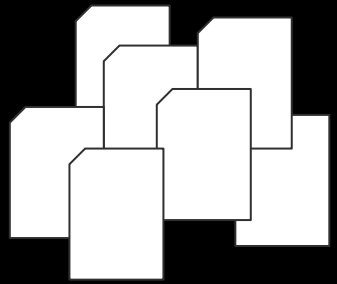
~ 52.34 hours

divide and conquer

+

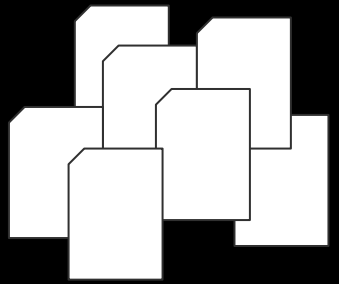
?

pages 1 - 700



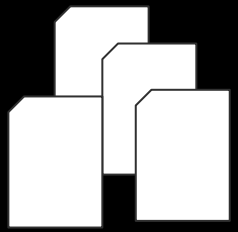
student 1

pages 701 - 1327



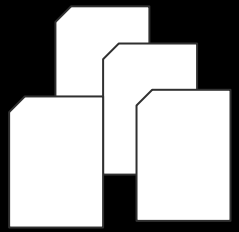
student 2

pages 1 - 350



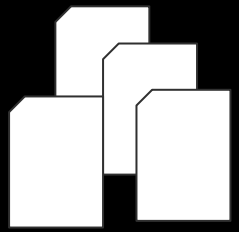
student 1

pages 351 - 700



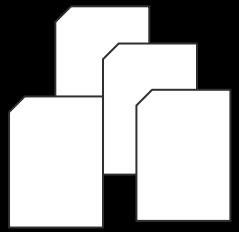
student 2

pages 701 - 1050



student 3

pages 1051- 1327

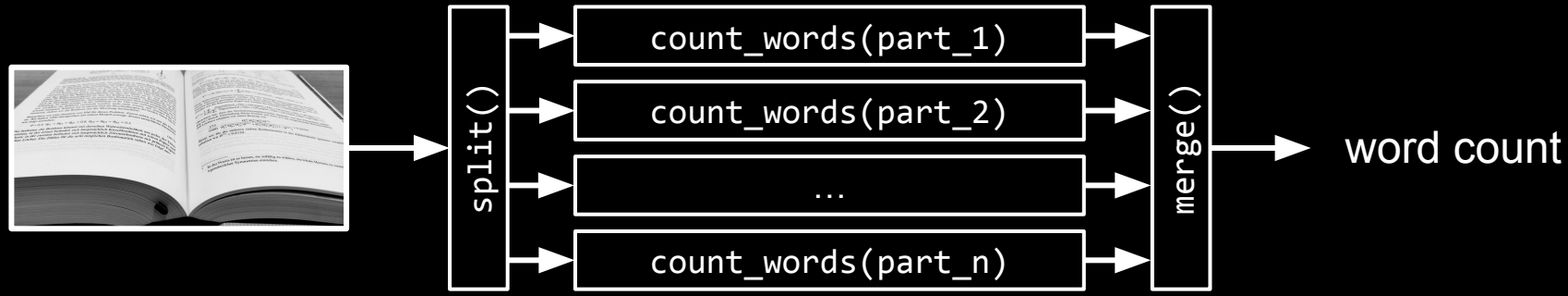


student 4

divide and conquer

+

distribution and parallelization


split()
count_words(part_1)
count_words(part_2)
...
count_words(part_n)
merge()
word count